v_0 is estimated to be about 4×10^{-13} erg. Using $2k_Fc\cong2\pi$, one finds that for two spins at a distance c apart, the interaction energy is roughly 12°K. The resistivity minimum occurs at about 12°K. Hence, it is safe to use the high-temperature expansion at around this temperature because the average nearest neighbor distance is much larger than c. One then finds that for $x=0.03\%$ the change in resistivity from 8 to 12°K is roughly

$$\Delta\rho=5\times10^{-13}\ \Omega\text{-cm}.$$

However, the observed variation is of the order of 10^{-10} Ω-cm.[16] This shows that the spin correlation effect does not explain the resistivity minimum phenomenon. At the present moment the resonant scattering theory seems to be more satisfactory because it is supported by another experiment.

PHYSICAL REVIEW VOLUME 132, NUMBER 2 15 OCTOBER 1963

Gap Equation and Current Density for a Superconductor in a Slowly Varying Static Magnetic Field

LUDWIG TEWORDT
Department of Physics, University of Notre Dame, Notre Dame, Indiana
(Received 10 June 1963)

The energy gap equation and the current density expression for a superconductor in a slowly varying static magnetic field are derived on the basis of a generalization of Nambu's Green's function formalism to finite temperatures. In the integral equation for the quasiparticle Green's function $G^A(\mathbf{R};\mathbf{r})$, expansions of G^A, the self-energy part Σ, and the vector potential $\mathbf{A}$, about the center-of-mass coordinates $\mathbf{R}$, are introduced. The integral equation is solved by iteration, and the contributions of all orders in the gap $\phi(\mathbf{R})$ are summed up. With the help of G^A, the generalized Ginzburg-Landau-Gor'kov (GLG) equations, valid at all temperatures for slowly varying $\mathbf{A}(\mathbf{R})$ and $\phi(\mathbf{R})$, are derived. For temperatures near T_c, correction terms to the coefficients of the GLG equations occur which are proportional to powers of $|\beta\phi|^2$. For temperatures near 0°K, the function multiplying the term $(\nabla+2ie\mathbf{A})^2\phi$ behaves like $\exp(-|\beta\phi|)$. The first-order correction to the term proportional to A^2 is found to be proportional to $\xi_0^2H^2$, for T near T_c and near 0°K ($\mathbf{H}$ = magnetic field strength, ξ_0 = coherence length). Our results are consistent with the formula of Nambu and Tuan for the reduction of the gap at 0°K in the London region.

I. INTRODUCTION

EQUATIONS for the superconducting energy gap in the presence of a magnetic field on the basis of the Bardeen-Cooper-Schrieffer[1] (BCS) and Bogoliubov microscopic theory have been derived by Gor'kov.[2] The validity of these equations is restricted to temperatures T, such that $T_c-T\ll T_c$, and to the local or London region where $q\xi_0\ll1$. Here T_c is the transition temperature, ξ_0 is the coherence length, and the q are the wave numbers of the field. By defining a wave function proportional to the energy gap, Gor'kov was able to transform his equations into the Ginzburg-Landau[3] phenomenological equations. In the following, the Gor'kov version of the Ginzburg-Landau equations is referred to as the GLG equations.

The GLG approach has been used to estimate the magnetic field dependence of the gap.[4] One finds good agreement between theory and experiment down to temperatures of about 0.7 T_c. However, one expects that at the lower temperatures corrections to the GLG equations will become significant. The aim of this paper is to generalize the GLG equations to all temperatures, under the assumption that the vector potential $\mathbf{A}(\mathbf{R})$ and the gap $\phi(\mathbf{R})$ vary slowly over the distance of a coherence length ξ_0. Our main concern is to establish the connection between the first generalized GLG equation and the equation of Nambu and Tuan[5] for the reduction of the energy gap at zero temperature in the local region. A characteristic point of their result is that the reduction of the gap depends only on the magnetic field strength.

Gor'kov has derived his equations with the help of integral equations for the quasiparticle Green's function. These integral equations were solved by iteration in powers of the gap, and only terms up to the fourth order in the gap were kept. This latter approximation is the origin of the restriction $T_c-T\ll T_c$. Our calculation is based on a generalization of Nambu's[6] two-component Green's function formalism to finite temperatures which has been developed in a previous paper.[7] Under the integral of the integral equation for

[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[2] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959) [translation: Soviet Phys.—JETP **9**, 1364 (1959)].
[3] V. L. Ginzburg and D. L. Landau, Zh. Eksperim. i Teor. Fiz. **20**, 1064 (1950).
[4] See, for instance, D. H. Douglass, Jr., Phys. Rev. Letters **6**, 346 (1961); **7**, 14 (1961); Phys. Rev. **124**, 735 (1961).
[5] Y. Nambu and S. F. Tuan, Phys. Rev. **128**, 2622 (1962).
[6] Y. Nambu, Phys. Rev. **117**, 648 (1960).
[7] L. Tewordt, Phys. Rev. **128**, 12 (1962).

the quasiparticle Green's function $G^A(\mathbf{R};\mathbf{r})$, we introduce expansions of the self-energy part Σ, the vector potential $\mathbf{A}$, and finally of G^A itself, about the center-of-mass coordinates $\mathbf{R}$. This procedure is valid since the Green's functions in the presence of a field, like the equilibrium Green's functions, are sharply peaked about the zero value of their relative coordinates $\mathbf{r}$, provided that the field varies slowly over the distance of a coherence length ξ_0. The integral equation is solved by iteration, and infinite summations over the contributions of all orders in the gap $\phi(\mathbf{R})$ are carried out. In this way one avoids the limitation to small gaps. The different contributions are ordered in powers of $\mathbf{A}(\mathbf{R})$, $\nabla\mathbf{A}(\mathbf{R})$, $\nabla\phi(\mathbf{R})$, $\nabla^2\phi(\mathbf{R})$, and combinations of these terms. In this paper we shall be concerned mainly with those terms which occur also in the GLG equations.

In Sec. II, the first generalized GLG equation is derived from the Hartree-Fock self-consistent equation for Σ, and, in Sec. III, the second generalized GLG equation is obtained from the expression for the current density in terms of G^A. In Sec. IV, the magnetic field-strength contribution is compared with the vector-potential contribution.

II. THE ENERGY GAP EQUATION

The analog of the first GLG equation will be derived from the Hartree-Fock self-consistent equation for the self-energy Σ of a quasiparticle in a magnetic field with vector potential $\mathbf{A}$. For finite temperatures this equation takes on the form

$$\Sigma_n(1,1')=\beta^{-1}\sum_{m=-\infty}^{+\infty}\tau_3 G_m{}^A(1,1')\tau_3 V_{n-m}(1-1'). \quad (2.1)$$

Here 1 and $1'$ refer to spatial coordinates, and the subscripts n, m, and $n-m$ refer to the imaginary energy variables $iE_n=i(2n+1)\pi\beta^{-1}$, etc., where $\beta=1/k_BT$ and n is an integer. The effective interaction potential between the electrons is denoted by V. The Fourier component $G_m{}^A$ of the thermodynamic Green's function satisfies the following equation of motion

$$\{iE_m+(2m)^{-1}\tau_3[\nabla_1-ie\mathbf{A}(1)\tau_3]^2+\mu\tau_3\}G_m{}^A(1,1') =\delta^3(1-1')+\int d2\,\Sigma_m(1,2)G_m{}^A(2,1'). \quad (2.2)$$

Here μ is the chemical potential. The Pauli spin matrices are denoted by τ_1, τ_2, and τ_3.

In order to obtain $G_m{}^A$ we rewrite the Eq. (2.2) as an integral equation,

$$G_m{}^A(1,1')=G_m{}^0(1-1')+\int d2\,d3\,G_m{}^0(1-2) \times[\Sigma_m(2,3)+K(2,3)]G_m{}^A(3,1'), \quad (2.3)$$

where $G_m{}^0$ satisfies

$$[iE_m+\tau_3(2m^{-1}\nabla_1{}^2+\mu)]G_m{}^0(1-1')=\delta^3(1-1'), \quad (2.4)$$

and where in the London gauge ($\mathrm{div}\mathbf{A}=0$) K can be written as

$$K(1,1')=(2m)^{-1}[ieA_i(1)(\nabla_1{}^i-\nabla_{1'}{}^i)+e^2A^2\tau_3] \times\delta^3(1-1'). \quad (2.5)$$

Since we are interested here in the local or London region, where $\mathbf{A}$ varies slowly in space, it is appropriate to consider $G_m{}^A(1.1')$, $\Sigma_m(1,1')$, and $K(1,1')$, as functions of the new variables $\mathbf{R}$ and $\mathbf{r}$ defined in terms of the old variables 1 and $1'$ by

$$\mathbf{R}=\tfrac{1}{2}(1+1'),\quad \mathbf{r}=1-1', \quad (2.6)$$

and to write $G_m{}^A=G_m{}^A(\mathbf{R};\mathbf{r})$, etc. In terms of these new variables Eq. (2.3) can be written as follows:

$$G_m{}^A(\mathbf{R};1-1')=G_m{}^0(1-1')+\int d2\,d3\,G_m{}^0(1-2) \times M_m[\mathbf{R}+\tfrac{1}{2}(2-1)+\tfrac{1}{2}(3-1');2-3] \times G_m{}^A[\mathbf{R}+\tfrac{1}{2}(3-2)+\tfrac{1}{2}(2-1);3-1']. \quad (2.7)$$

Here we have introduced the abbreviation

$$M_m=\Sigma_m+K. \quad (2.8)$$

Since G^0, M, and G^A in the integrand of Eq. (2.7) are each sharply peaked about the value zero of their respective $\mathbf{r}$ variables, we can consider all the coordinate differences $(2-1)$, $(3-1')$, and $(3-2)$ to be small. We expand now M and G^A in a Taylor series in their first arguments about $\mathbf{R}$, and neglect higher than second-order terms in the coordinate differences. Then we insert the Fourier transforms of G^0, M, and G^A in their respective $\mathbf{r}$ variables; these will be denoted by $G^0(\mathbf{p})$, $M(\mathbf{R};\mathbf{p})$, and $G^A(\mathbf{R};\mathbf{p})$. Finally, we express factors $(2-1)$ and $(3-1')$ in terms of derivatives of $G^0(\mathbf{p})$ and $G^A(\mathbf{R};\mathbf{p})$ with respect to $\mathbf{p}$. We neglect the expansion terms with factors $(3-2)$ which would lead to terms $\nabla_pM=\nabla_p\Sigma+\nabla_pK$; this is justified since $\Sigma(\mathbf{R};\mathbf{p})$ does not depend on $\mathbf{p}$ for the model potential of BCS which we shall use, and since the terms arising from ∇_pK turn out to vanish. In this way we obtain the following expression for $G_m{}^A$ (occasionally for shortness we leave out the subscripts m and the arguments $\mathbf{R}$ and $\mathbf{p}$; no confusion is possible since all terms refer to the same m, $\mathbf{R}$, and $\mathbf{p}$)

$$G_m{}^A(\mathbf{R};\mathbf{p})=G^0+G^0MG^A-\tfrac{1}{2}i(\nabla_p{}^iG^0)[(\nabla_R{}^iM)G^A+M(\nabla_R{}^iG^A)]+\tfrac{1}{2}iG^0(\nabla_R{}^iM)(\nabla_p{}^iG^A)$$
$$-\tfrac{1}{8}(\nabla_p{}^i\nabla_p{}^jG^0)[(\nabla_R{}^i\nabla_R{}^jM)G^A+M(\nabla_R{}^i\nabla_R{}^jG^A)]-\tfrac{1}{8}G^0(\nabla_R{}^i\nabla_R{}^jM)(\nabla_p{}^i\nabla_p{}^jG^A)+\tfrac{1}{4}(\nabla_p{}^iG^0)(\nabla_R{}^i\nabla_R{}^jM)(\nabla_p{}^jG^A)$$
$$-\tfrac{1}{4}(\nabla_p{}^i\nabla_p{}^jG^0)(\nabla_R{}^iM)(\nabla_R{}^jG^A)+\tfrac{1}{4}(\nabla_p{}^jG^0)(\nabla_R{}^iM)(\nabla_p{}^i\nabla_R{}^jG^A). \quad (2.9)$$

Here ∇_R^i and ∇_p^i mean derivatives with respect to R_i and p_i, respectively, and M is, according to Eqs. (2.8) and (2.5), equal to

$$M_m(\mathbf{R};\mathbf{p})=\Sigma_m(\mathbf{R};\mathbf{p})+K(\mathbf{R};\mathbf{p}), \tag{2.10}$$

with

$$K(\mathbf{R};\mathbf{p})=-eA_i(\mathbf{R})(p_i/m). \tag{2.11}$$

The term with A^2 in K has been left out since, according to our calculations, it does not give a contribution. The propagator G^0 is determined from Eq. (2.4) to be

$$G_m^0(\mathbf{p})=(iE_m-\epsilon_p\tau_3)^{-1}, \tag{2.12}$$

with $\epsilon_p=[(p^2/2m)-\mu]$. From Eq. (2.12) we find that

$$\nabla_p^iG_m^0(\mathbf{p})=G_m^0(\mathbf{p})(p_i/m)\tau_3G_m^0(\mathbf{p}). \tag{2.13}$$

We now solve Eq. (2.9) for G_m^A by iteration and keep only terms up to second order in K. If we carry out partial summations over all the terms containing only factors G^0 and Σ, then we obtain propagators G equal to

$$G_m(\mathbf{R};\mathbf{p})=[G_m^{0-1}(\mathbf{p})-\Sigma_m(\mathbf{R};\mathbf{p})]^{-1}, \tag{2.14}$$

and in terms of these the result turns out to be

$$\begin{aligned}G_m^A(\mathbf{R};\mathbf{p})=G+GKG+GKGKG+\frac{ip_i}{2m}\{G(\nabla_iM)G\tau_3G-G\tau_3G(\nabla_iM)G\}+\frac{ip_i}{2m}\{G(\nabla_iM)G\tau_3GKG\\ -G\tau_3G(\nabla_iM)GKG+GKG(\nabla_iM)G\tau_3G-GKG\tau_3G(\nabla_iM)G+G(\nabla_iM)GKG\tau_3G-G\tau_3GKG(\nabla_iM)G\}\\ -\frac{p_ip_j}{4m^2}\{G(\nabla_i\nabla_jM)G\tau_3G\tau_3G+G\tau_3G\tau_3G(\nabla_i\nabla_jM)G-G\tau_3G(\nabla_i\nabla_jM)G\tau_3G\}\\ +\frac{p_ip_j}{2m^2}\{G(\nabla_iM)G\tau_3G\tau_3G(\nabla_jM)G+G\tau_3G(\nabla_iM)G(\nabla_jM)G\tau_3G\\ -G(\nabla_iM)G(\nabla_jM)G\tau_3G\tau_3G-G\tau_3G\tau_3G(\nabla_iM)G(\nabla_jM)G\}.\end{aligned} \tag{2.15}$$

Here, and later on, ∇_i denotes the derivative with respect to R_i. For shortness we have left out all terms of the form $(p_ip_j/m^2)G\tau_3G\tau_3G(\nabla_iM)G(\nabla_jM)G$ in this section.

We determine now the τ_1 and τ_2 components of G_m^A by multiplying Eq. (2.15) by τ_1 or τ_2, respectively, and taking the trace of the resulting equation. Since we are interested here only in the energy gap terms and not in the renormalization terms of Σ, we set

$$\Sigma(\mathbf{R};\mathbf{p})=\phi_1(\mathbf{R})\tau_1+\phi_2(\mathbf{R})\tau_2. \tag{2.16}$$

It is sufficient to keep, in the case of the self-energy equation, only the terms with an even number of factors p_i in the expression for G_m^A in Eq. (2.15), since the odd terms drop out when we integrate later over d^3p. Then we find

$$\begin{aligned}(G_m^A)_{\tau_1}&=(G)_{\tau_1}+(p_ip_j/m^2)[a_1e^2A_iA_j-b(\tfrac{1}{2}i\nabla_i\phi_2)eA_j+c_1(-\tfrac{1}{4}\nabla_i\nabla_j\phi_1)+d(-\tfrac{1}{4}\nabla_i\nabla_j\phi_2)],\\ (G_m^A)_{\tau_2}&=(G)_{\tau_2}+(p_ip_j/m^2)[a_2e^2A_iA_j+b(\tfrac{1}{2}i\nabla_i\phi_1)eA_j+c_2(-\tfrac{1}{4}\nabla_i\nabla_j\phi_2)+d(-\tfrac{1}{4}\nabla_i\nabla_j\phi_1)],\end{aligned} \tag{2.17}$$

where a_k, c_k $(k=1,2)$, and b, d, are given by the expressions

$$\begin{aligned}a_k&=\tfrac{1}{2}\operatorname{Tr}(GGG\tau_k)=\tfrac{1}{4}(\partial/\partial\phi_k)\operatorname{Tr}(GG),\\ b&=\tfrac{1}{2}\operatorname{Tr}(G\tau_1G\tau_2G\tau_3G-G\tau_1G\tau_3G\tau_2G+G\tau_2G\tau_3G\tau_1G-G\tau_3G\tau_2G\tau_1G+G\tau_3G\tau_1G\tau_2G-G\tau_2G\tau_1G\tau_3G),\\ c_k&=\tfrac{1}{2}\operatorname{Tr}(2G\tau_kG\tau_3G\tau_3G\tau_k-G\tau_3G\tau_kG\tau_3G\tau_k),\\ d&=\tfrac{1}{2}\operatorname{Tr}(G\tau_2G\tau_3G\tau_3G\tau_1+G\tau_3G\tau_3G\tau_2G\tau_1-G\tau_3G\tau_2G\tau_3G\tau_1).\end{aligned} \tag{2.18}$$

We now make use of these results for G_m^A and insert them into the Fourier transform of the self-energy equation (2.1) in the $\mathbf{r}$ variable; that is,

$$\tau_3\Sigma_n(\mathbf{R};\mathbf{l})\tau_3=\int\frac{d^3p}{(2\pi)^3}\beta^{-1}\sum_{m=-\infty}^{+\infty}G_m^A(\mathbf{R};\mathbf{p})V_{n-m}(\mathbf{l}-\mathbf{p}). \tag{2.19}$$

We calculate the τ_1 and τ_2 components of Eq. (2.19) by inserting the expression (2.16) for Σ on the left and the components (2.17) of G_m^A on the right-hand side of this equation. Then we add i times the τ_2 components to the

τ_1 component, where i is the imaginary unit, and introduce the complex energy gap ϕ by

$$\phi(\mathbf{R})=\phi_1(\mathbf{R})+i\phi_2(\mathbf{R}). \tag{2.20}$$

The result of this procedure is

$$\phi=-\int\frac{d^3p}{(2\pi)^3}\bar{V}\beta^{-1}\sum_m\Big\{\phi N_m^{-1}+\frac{p_ip_j}{m^2}[(a_1+ia_2)e^2A_iA_j+ib(\tfrac{1}{2}i\nabla_i\phi)eA_j+\tfrac{1}{2}(c_1+c_2)(-\tfrac{1}{4}\nabla_i\nabla_j\phi)+[\tfrac{1}{2}(c_1-c_2)+id](-\tfrac{1}{4}\nabla_i\nabla_j\phi^*)]\Big\}. \tag{2.21}$$

Here $\bar{V}$ is an average of the potential, and G_m is, according to Eqs. (2.14), (2.12), and (2.16), equal to

$$G_m(\mathbf{R};\mathbf{p})=(iE_m+\epsilon_p\tau_3+\phi_1\tau_1+\phi_2\tau_2)N_m^{-1}, \tag{2.22}$$

where N_m is given by

$$N_m=(iE_m)^2-E^2,\quad E=(\epsilon_p^2+|\phi|^2)^{1/2}. \tag{2.23}$$

E is the energy of a quasiparticle in the presence of the field. If we insert the expression Eq. (2.22) for G into the Eqs. (2.18), we find

$$a_1+ia_2=(\partial/\partial\phi^*)\tfrac{1}{2}\operatorname{Tr}(G_mG_m),\quad b=2i\phi^{-1}(a_1+ia_2), \tag{2.24}$$

$$\tfrac{1}{2}(c_1+c_2)=-\tfrac{1}{2}ib-2|\phi|^2N_m^{-3}(1-2\epsilon_p^2N_m^{-1}), \tag{2.25}$$

$$\tfrac{1}{2}(c_1-c_2)+id=2\phi^2N_m^{-3}(1+2\epsilon_p^2N_m^{-1}). \tag{2.26}$$

We now introduce the expressions Eqs. (2.24) and (2.25) into Eq. (2.21) and write the resulting equation as follows:

$$\Big\{\tfrac{2}{3}\epsilon_FN\bar{V}\Big[\phi^{-1}\frac{\partial}{\partial\phi^*}\int_0^\omega d\epsilon\beta^{-1}\sum_m\tfrac{1}{2}\operatorname{Tr}(G_mG_m)\Big](2m)^{-1}(\boldsymbol{\nabla}+2ie\mathbf{A})^2-\Big[1+2N\bar{V}\int_0^\omega d\epsilon\,\beta^{-1}\sum_m N_m^{-1}\Big]\Big\}\phi=0, \tag{2.27}$$

where $N=mp_F/2\pi^2$ is the density of states, $\epsilon_F=p_F^2/2m$ is the Fermi energy, and ω is the cutoff frequency for the effective potential which is of the order of the Debye frequency. In Eq. (2.27) we have left out for shortness the contributions which arise from the second term in Eq. (2.25) and the term in Eq. (2.26).

The second term in square brackets in Eq. (2.27) turns out to be

$$\Big[1+2N\bar{V}\int_0^\omega d\epsilon\,\beta^{-1}\sum_m N_m^{-1}\Big]=1-N\bar{V}\int_0^\omega d\epsilon\,E^{-1}\tanh(\tfrac{1}{2}\beta E)=N\bar{V}[g_1(|\beta\phi|)-g_1(\beta\phi_0)]. \tag{2.28}$$

The last line has been obtained with the help of a relation derived by Bardeen[8] [see Eq. (4.7) in Ref. 8]. Expansions of the function $g_1(x)$, for $x<1$ and $x>1$, are given in Ref. 8. The notation ϕ_0 is used for the energy gap at temperature T in the absence of fields. ϕ_0 is taken to be real.

The sum over m in the first term in square brackets of Eq. (2.27) can be carried out by means of contour integration in the complex iE_m plane.[7] It may be remarked that the sum over m in Eq. (2.28) has been determined by the same method. We find (η is a positive infinitesimal)

$$\beta^{-1}\sum_{m=-\infty}^{+\infty}\tfrac{1}{2}\operatorname{Tr}(G_mG_m)=\pi^{-1}\int_{-\infty}^{+\infty}\frac{dx}{(1+e^{-\beta x})}\operatorname{Im}\Big\{\frac{x^2+E^2}{[x^2-E^2+i\eta x]^2}\Big\}=-\Big\{\frac{d}{dx}\Big[\frac{(x^2+E^2)}{(x+E)^2}\tanh\tfrac{1}{2}\beta x\Big]\Big\}_{x=E}=-\frac{\beta e^{\beta E}}{(1+e^{\beta E})^2}. \tag{2.29}$$

In the weak-coupling limit the ϵ integral over the right-hand side of Eq. (2.29) becomes equal to the expression $\frac{1}{2}[(\Lambda/\Lambda_T)-1]$ in BCS [see Eq. (5.24) in Ref. 2]. It is more convenient to use another equivalent expression for (Λ/Λ_T) given by BCS [see Eq. (C19) in Ref. 2]. By using the aforementioned relation of Bardeen,[8] we can express (Λ/Λ_T) in terms of the derivative of the function $g_1(|\beta\phi|)$ with respect to ϕ. In this way we obtain for the first

[8] J. Bardeen, Rev. Mod. Phys. 34, 667 (1962).

term in square brackets in Eq. (2.27), denoted by $\gamma(|\beta\phi|)$, the relation

$$\gamma(|\beta\phi|)=\tfrac{1}{2}\phi^{-1}\frac{\partial}{\partial\phi^*}\left(\frac{\Lambda}{\Lambda_T}\right)=\frac{\partial}{\partial\phi^*}\left[\frac{\partial}{\partial\phi}g_1(|\beta\phi|)\right]. \tag{2.30}$$

Making use of the abbreviation γ and Eq. (2.28), Eq. (2.27) becomes

$$\left\{\frac{\gamma(|\beta\phi|)}{2m}[\nabla+2ie\mathbf{A}(\mathbf{R})]^2+\left(\frac{3}{2\epsilon_F}\right)[g_1(\beta\phi_0)-g_1(|\beta\phi|)]\right\}\phi=0. \tag{2.31}$$

In the temperature range near T_c, i.e., for $(\beta\phi_0)<1$, one can use the power series $g_1(x)=a_2x^2+a_4x^4+\cdots$, given in Ref. 8, and with the help of Eq. (2.30), Eq. (2.31) goes over into the following equation:

$$\{[1+4(a_4/a_2)|\beta\phi|^2+\cdots](2m)^{-1}(\nabla+2ie\mathbf{A})^2+(3/2\epsilon_F)[1+(a_4/a_2)(|\beta\phi|^2+(\beta\phi_0)^2)+\cdots](\phi_0^2-|\phi|^2)\}\phi=0. \tag{2.32}$$

The dots denote higher order terms in $|\beta\phi|^2$ and $(\beta\phi_0)^2$. In the lowest order approximation Eq. (2.32) becomes identical to the first GLG equation.

In the temperature range near 0°K, i.e., for $(\beta\phi_0)\gg1$, one finds with the help of Eq. (2.30) and the appropriate expansion for g_1, given in Ref. 8, that to a lowest order approximation Eq. (2.31) becomes

$$\{(\pi/8)^{1/2}\beta^2(|\beta\phi|)^{-1/2}\exp(-|\beta\phi|)(2m)^{-1}(\nabla+2ie\mathbf{A})^2+(3/2\epsilon_F)\ln(\phi_0/|\phi|)\}\phi=0. \tag{2.33}$$

III. THE CURRENT DENSITY

The analog of the second GLG equation will be derived from the expression for the current density in terms of the Green's function $G_m{}^A$ for the quasiparticle in the presence of the field. The paramagnetic part of the current density, denoted by $\mathbf{j}^p$, is given by

$$\mathbf{j}^p(\mathbf{R})=\frac{ie}{2m}\beta^{-1}\sum_{m=-\infty}^{+\infty}\{(\nabla_1-\nabla_{1'})\operatorname{Tr}(G_m{}^A(1,1'))\}_{1=1'=R}=-\frac{2e}{m}\int\frac{d^3p}{(2\pi)^3}p_i\beta^{-1}\sum_m\tfrac{1}{2}\operatorname{Tr}(G_m{}^A(\mathbf{R};\mathbf{p})). \tag{3.1}$$

Now only the terms with an odd number of factors p_j in the expression for $G_m{}^A$, Eq. (2.15), will contribute to the integral over d^3p in Eq. (3.1). The contribution due to GKG together with the diamagnetic part of the current density, that is, $\mathbf{j}^d(\mathbf{R})=-\Lambda^{-1}\mathbf{A}(\mathbf{R})$, yields the BCS expression for the current density in the London limit which will be denoted by $\mathbf{j}^0$. This contribution is given by

$$\mathbf{j}^0(\mathbf{R})=-\Lambda_T{}^{-1}\mathbf{A}(\mathbf{R}). \tag{3.2}$$

In addition, we have to take into account the correction to the paramagnetic part which is due to the change of the gap in space. This correction is determined by the first term in curled brackets in the expression for $G_m{}^A$ in Eq. (2.15), and it will be denoted by $\Delta\mathbf{j}^p$. We obtain

$$\Delta j_i{}^p(\mathbf{R})=\frac{ie}{m^2}\int\frac{d^3p}{(2\pi)^3}p_ip_j\beta^{-1}\sum_m[\tfrac{1}{2}\operatorname{Tr}(G\tau_3G\tau_1G-G\tau_1G\tau_3G)\nabla_j\phi_1+\tfrac{1}{2}\operatorname{Tr}(G\tau_3G\tau_2G-G\tau_2G\tau_3G)\nabla_j\phi_2]. \tag{3.3}$$

By using Eq. (2.22) for G, we find

$$\begin{aligned}\tfrac{1}{2}\operatorname{Tr}(G\tau_3G\tau_1G-G\tau_1G\tau_3G)&=2i\phi_2N_m{}^{-2},\\ \tfrac{1}{2}\operatorname{Tr}(G\tau_3G\tau_2G-G\tau_2G\tau_3G)&=-2i\phi_1N_m{}^{-2}.\end{aligned} \tag{3.4}$$

If we insert these results into the Eq. (3.3) and introduce the complex gap ϕ, as defined in Eq. (2.20), we obtain

$$\Delta\mathbf{j}^p(\mathbf{R})=-\left(\frac{ie}{m}\right)(\phi^*\nabla\phi-\phi\nabla\phi^*)n\int_0^\infty d\epsilon\,\beta^{-1}\sum_m N_m{}^{-2}, \tag{3.5}$$

where n is the electron number density. The summation over m can be carried out by contour integration as before,[7] and the result of the $d\epsilon$ integration can be expressed in terms of (Λ/Λ_T) [see BCS, Eq. (C19)]:

$$\int_0^\infty d\epsilon\,\beta^{-1}\sum_m N_m{}^{-2}=-\frac{1}{4}\int_0^\infty\frac{d\epsilon}{E}\frac{d}{dE}\left(\frac{\tanh\frac{1}{2}\beta E}{E}\right)=\frac{1}{4}\frac{\Lambda}{\Lambda_T}|\phi|^{-2}. \tag{3.6}$$

From the Eqs. (3.5), (3.6), and (3.2), we find for the total current density $\mathbf{j}=\mathbf{j}^0+\Delta\mathbf{j}^p$:

$$\mathbf{j}(\mathbf{R})=\left\{-\frac{ie}{m}(\phi^*\nabla\phi-\phi\nabla\phi^*)-\frac{4e^2}{m}|\phi|^2\mathbf{A}\right\}\frac{n\Lambda}{4\Lambda_T|\phi|^2}. \quad (3.7)$$

For temperatures near T_c one obtains, with the help of the relation between (Λ/Λ_T) and g_1 [see Eq. (2.30)] and the power series for g_1,

$$(\Lambda/\Lambda_T)|\phi|^{-2}=2a_2\beta^2[1+2(a_4/a_2)|\beta\phi|^2+\cdots]. \quad (3.8)$$

The coefficient a_2 is equal to $7\zeta(3)/8\pi^2$ [$\zeta(x)$ is Riemann's zeta function]. If we insert Eq. (3.8) into Eq. (3.7) and neglect the correction terms in $|\beta\phi|^2$, we obtain exactly the second GLG equation. In the zero-temperature limit (Λ/Λ_T) approaches one.

IV. CONCLUSION

Our generalized Ginzburg-Landau-Gor'kov (GLG) equations for the energy gap $\phi(\mathbf{R})$ in the presence of a magnetic field with vector potential $\mathbf{A}(\mathbf{R})$ are given by Eqs. (2.31) and (3.7). The function $g_1(|\beta\phi|)$ has been introduced by Bardeen,[8] and the function $\gamma(|\beta\phi|)$ is connected to g_1 through the relation $\gamma=(\partial/\partial\phi^*)\times[(\partial/\partial\phi)g_1]$ [see Eq. (2.30)]. These equations have been derived under the assumptions that $\phi(\mathbf{R})$ and $\mathbf{A}(\mathbf{R})$ vary slowly over the distance of a coherence length ξ_0. In the high-temperature region, i.e., for $(\beta\phi_0)<1$, Eqs. (2.31) and (3.7) go over into Eqs. (2.32) and (3.7), combined with (3.8). Here ϕ_0 denotes the gap in the absence of fields. In these equations correction terms to the original GLG equations[1] occur, which are proportional to powers in $|\beta\phi|^2$ and $(\beta\phi_0)^2$. In the low-temperature region, more exactly, for $|\beta\phi|>1$, Eq. (2.31) takes on the form of Eq. (2.33). In this equation the function multiplying the term $(\nabla+2ie\mathbf{A})^2\phi$ behaves like $\exp(-|\beta\phi|)$ and therefore vanishes in the zero-temperature limit. However, the second term in Eq. (2.25) and the term in Eq. (2.26) will give rise to additional contributions to Eq. (2.21), and thus to Eq. (2.31), which are proportional to $\nabla^2\phi$ and $\nabla^2\phi^*$, respectively. It turns out that the coefficients of these terms stay finite in the zero temperature limit.

Since the contributions proportional to A^2 and $\mathbf{A}\cdot\nabla\phi$ in our first generalized GLG equation drop out in the zero-temperature limit, we have to take into account also the terms containing five G's in our expression Eq. (2.15) for the propagator G^A. These terms lead to contributions proportional to $(\nabla\phi)^2$ and H^2 {we use H^2 as an abbreviation for $[(\nabla_iA_j)^2+(\nabla_iA_j)\times(\nabla_jA_i)+(\nabla_iA_i)^2]$}. So far, we have investigated only the H^2 term, and we have compared it with the A^2 term. The additional term on the left-hand side of Eq. (2.31) is easily calculated from Eqs. (2.15) and (2.19) and found to be

$$+\frac{v_F^2e^2}{15m}\left[\frac{\partial^2}{\partial(|\phi|^2)^2}\left(1+|\phi|^2\frac{\partial}{\partial|\phi|^2}\right)g_1(|\beta\phi|)\right]\times[(\nabla_iA_j)^2+(\nabla_iA_j)(\nabla_jA_i)+(\nabla_iA_i)^2]\phi. \quad (4.1)$$

From Eqs. (4.1) and (2.31) we derive that in the temperature region near T_c the ratio of the H^2 contribution to the A^2 contribution becomes $-\frac{1}{5}(a_4/a_2)\times(v_F\beta)^2(H^2/A^2)$. Therefore, this ratio becomes small in the local region. However, for $|\beta\phi|\gg1$, the ratio of the H^2 contribution to the A^2 contribution is found to be of the order of magnitude $(v_F/|\phi|)^2(|\beta\phi|)^{-3/2}\times\exp(|\beta\phi|)(H^2/A^2)$, and therefore the H^2 contribution becomes predominant in comparison to the A^2 contribution. In the zero-temperature limit Eq. (2.31), together with Eq. (4.1), (and $\nabla\phi$ set equal to zero) leads to the formula of Nambu and Tuan[5] for the reduction of the gap.